Name____Date______________Period______

AFM Final Exam Review: Part 2 (Standards 2.01, 2.02, 2.03, 2.04, and 2.05)

2.01 Use logarithmic (common, natural) functions to model and solve problems; justify results.

a. Solve using tables, graphs, and algebraic properties.

b. Intepret the constants, coefficients, and bases in the context of the problem.

Recall from the carbon dating investigation: As long as the animal is alive, the ratio of C-14 to C-12 in its body remains constant. After the animal dies, carbon-14 continues to decay without being replaced. The current amount of radioactive Carbon-14 present in the remains of animal bones can be measured, and the ratio of the current amount of Carbon-14 to its initial amount can be used to determine age.

1. The ratio C of the current amount of Carbon-14 to its initial amount is described by the function

$$C(t) = 0.5^{\frac{t}{5730}},$$

where t is the time (in years) since the living thing perished.

a. What does the number 5,730 represent in the equation?

5,730 years is the half-life of Carbon-14. If you plug in 5750 for t, C = 0.5.

b. How much Carbon-14 remains in a 2500-year-old mummy?

$C(2500) = 0.5^{\frac{2500}{5730}} = 0.739$

73.9% of the Carbon-14 remains.

c. An animal bone is found that has 30% Carbon-14 remaining. How old is the bone?

$30\% = 0.3 \Rightarrow 0.3 = 0.5^{\frac{t}{5730}}$

$\frac{\log 0.3}{\log 0.5} = \frac{t}{5730}$, $t \approx 9953$ years.

Where you should go for more review on this concept: Carbon Dating Problem (12/11/12 post); Logs and Exponentials Test and Test Review (key to review online; see 12/12/12 post).

2. The population of Mozambique is growing at a rate of 3.36% per year and currently has a population of 25 million people.

a. Write an exponential function *P(t)* that models Mozambique's population *P* in millions as a function of time *t* in years.

$y = ab^x$

$a = 25$ million

$b = 1 + 0.0336$

$P(t) = 25(1.0336)^t$

b. How long will it take for the population of that country to double?

$50 = 25(1.0336)^t$

$2 = 1.0336^t$

$\log 2 = \log 1.0336^t$

$\log 2 = t \cdot \log 1.0336$

$t = \frac{\log 2}{\log 1.0336}$

$t \approx 21$ years.

c. The population of Bulgaria is **declining** at a rate of 2.01% per year. How long will it take for that country's population to drop to 60% of its current level?

$a = 1$

$b = 1 - .0201$

$= .9799$

$0.6 = 1(.9799)^t$

$\frac{\log 0.6}{\log .9799} = t \approx 25$ years

Where you should go for more review on this concept: Logs-Population Growth Problem (12/7/12 post); Logs and Exponentials Test and Test Review (key to review online; see 12/12/12 post).

3. Recall the formula for continuously compounding interest, $A = Pe^{rt}$ where P is the principal (initial deposit), r is the interest rate (as a decimal), and A is the balance after t years.

a. If you deposit \$10,000 into an account with a 5% interest rate compounded continuously, how much will be in the account after 15 years?

$P = 10{,}000$

$r = 0.05$

$t = 15$

$A = 10{,}000e^{0.05 \cdot 15}$

$\approx \$21{,}170$

b. How long will it take for the money in the account to reach \$20,000?

$20{,}000 = 10{,}000e^{0.05 \cdot t}$

$2 = e^{0.05 \cdot t}$

$\ln 2 = \ln e^{0.05t}$

$\ln 2 = 0.05t$

$t = 13.86$ years

OR: Solve by graphing

Where you should go for more review on this concept: Compound Interest Investigation (11/16/12 post); Compound Interest Quiz and Quiz Review (key to review online; see 11/27/12 post that includes a helpful video).

2.02 Use piecewise-defined functions to model and solve problems; justify results.

a. Solve using tables, graphs, and algebraic properties.

b. Interpret the constants, coefficients, and bases in the context of the problem.

4. The following function models a monthly cellphone bill B in dollars as a function of the time t in minutes that the customer has used the phone.

$$B(t) = \begin{cases} 0.10t + 12, & 0 \le t < 30 \\ 0.08(t - 30) + 15, & 30 \le t < 150 \\ 0.02(t - 150) + 24.6, & 150 \le t < 600 \end{cases}$$

a. How much is the bill if you talk on the phone for 130 minutes?

Use Step 2: $B(t) = 0.08(130-30)+15$

$= \$23$

b. What does the slope represent in the first step of the function?

You will pay 10¢/minute if you talk for less than 30 mins per month.

c. What does the y-intercept represent in the first step of the function?

You get charged a flat rate of \$12 if you talk for less than 30 mins per month.

d. How many minutes have you used the phone if your bill is \$17.00?

You must find an intersection w/ $Y=17$ that's in the domain.

$Y1 = $ Step 2 Intersection: $(55, 17)$

$Y2 = 15$

55 minutes

e. What is the domain for the $B(t)$ function? What is the range?

Domain: $0 \le t < 600$

Range: $B(0) \le B(t) < B(600)$

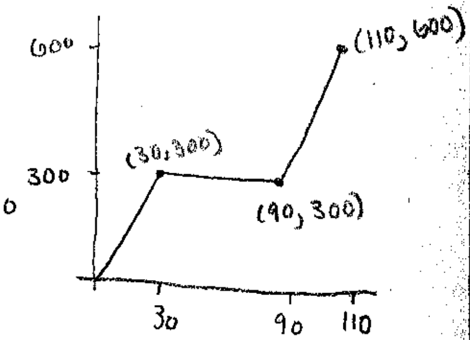
$= 0 \le B(t) < 33.6$

Where you should go for more review on this concept: MSL Constructed Response Sample Questions (3/25/13 post); various warm ups; the Olympic Swimming Times problem (9/24/12 post)

5. A car drives at a constant rate of 10 ft/second for 30 seconds, then stops at a red light for 60 seconds. The light turns green and the car drives at a rate of 15 ft/sec until it arrives at its destination 20 seconds later.

a. Write a piecewise function *D(t)* that models the car's distance from its starting point *D* is a function of time *t* in seconds. Be sure you state the domain for each step of the function.

$$D(t) = \begin{cases} 10t, & 0 \le t < 30 \\ 300, & 30 \le t < 90 \\ 15(t-90)+300 & 90 \le t \le 110 \end{cases}$$



b. How far has the car traveled after 100 seconds?

$15(100-90)+300 = 450$ ft

c. How long will it take for the car to be 400 feet from its starting point?

$400 = 15(t-90)+300$

$100 = 15(t-90)$

$t = 96.67$ seconds

d. What is the domain of the *D(t)* function? What is the range?

Domain: $0 \le t \le 110$

Range: $0 \le D(t) \le 600$

Where you should go for more review on this concept: In addition to the suggestions below #4, see the Balloon Rocket Project (9/19/12 post) to understand the relationship between slope and speed.

2.03 Use power functions to model and solve problems; justify results.

a. Solve using tables, graphs, and algebraic properties.

b. Interpret the constants, coefficients, and bases in the context of the problem.

6. The time T it takes to travel 100 miles depends on your speed s.

a. Write a function $T(s)$ that models the time T in hours it takes to go 100 miles as a function of the speed s in miles per hour.

$T(s) = \frac{100}{s} = 100s^{-1}$

b. How long will it take you to go 100 miles if you're traveling 15 miles per hour?

$T(15) = \frac{100}{15} = 6.67$ hours

c. How fast must you go if you need to go 100 miles in 90 minutes? 90 minutes = 1.5 hours

$1.5 = \frac{100}{s}$

$1.5s = 100$

$s = 66.67$ miles/hr

d. What is the practical domain for the $T(s)$ function in the context of this problem? What is the practical range?

Practical Domain:

$0 < s <$ a very high speed

Practical Range:

$0 < T(s) <$ a very long time

e. What is the theoretical domain for the $T(s)$ function? The theoretical range?

Domain: $0 < s$ and $0 > s$ $(s \neq 0)$

Range: $0 < T(s)$ and $0 > T(s)$ $(T(s) \neq 0)$

Where you should go for more review on this concept: See the Comparing Power Functions Investigation (3/19/13 post) to help you understand domain and range; see also various power function warm ups from around that time.

7. The period P in seconds of a pendulum depends on its length ℓ in centimeters. This relationship can be modeled by the power function

$$P(\ell) = 0.2\ell^{0.51}.$$

a. Is this function closest to a quadratic function, a square root function, or an inverse variation function?

Square root function $(\sqrt{x} = x^{\frac{1}{2}})$

b. What does the number 0.2 mean in the context of the problem?

When the length is 1 cm, the period is 0.2 seconds $(P(1) = 0.2)$.

c. If a pendulum is 1 m (100 cm) long, what is its period?

$P(100) = 0.2(100)^{0.51} = 2.09$ seconds

d. Suppose you want to make your pendulum into a clock so that its period is exactly 2 seconds. How many centimeters long should the pendulum be?

$Y1 = 0.2(100)^{0.51}$ Intersect = (91.4, 2)

$Y2 = 2$ The pendulum should be 91.4 cm long.

e. Suppose you only have 10 m of string to make a pendulum and therefore 10 m is the maximum length it could be. What is the practical domain for the $P(\ell)$ function in context? What is the practical range? 10 m = 1000 cm

Practical Domain: $0 < \ell \leq 1000$

Practical Range: $P(0) < P(\ell) \leq P(1000)$

$= 0 < P(\ell) \leq 6.78$ seconds

f. What is the theoretical domain and range for the $P(\ell)$ function?

Domain: $0 < \ell$

Range: $0 < P(\ell)$

Where you should go for more review on this concept: See the Pendulum Lab (3/21/13 post).

2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.

a. Solve using tables, graphs, and algebraic properties.

b. Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.

8. The number of hours of daylight in Dallas, TX can be modeled by the function

$$d(t) = 2.36 \sin\left(\frac{2\pi}{365}t\right) + 12.11,$$

where d is the number of hours of daylight and t is the number of days that have passed since the Spring Equinox (around March 21).

a. Identify the following features of the function:

Amplitude: 2.36 Vertical Shift: 12.11

Frequency: $\frac{2\pi}{365}$ Period: $\frac{2\pi}{\left(\frac{2\pi}{365}\right)} = 365$

b. What does the period mean in the context of this problem?

The cycle repeats every 365 days, or each year.

c. How long is the longest day of the year in Dallas? How many days after the Spring Equinox does it occur?

Calculate maximum: (91.25, 14.47)

The longest day is 14.47 hours and it will occur 91 days after the Spring Equinox.

d. How long is the shortest day of the year in Dallas? How many days after the Spring Equinox does it occur?

Calculate minimum: (273.75, 9.75)

The shortest day is 9.75 hours, and it will occur 274 days after the Spring Equinox.

e. How many hours of daylight will there be in Dallas 45 days after the Spring Equinox?

$d(45) = 13.761$ hrs

Table:

x	y
45	13.761

f. How many days after the Spring Equinox will the number of daylight hours **first** reach 13?

Y1 = d(t) Find the first intersection: (22.5, 13)

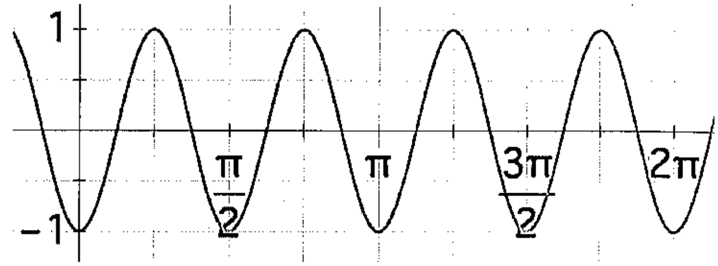
Y2 = 13 22.5 days after the Equinox

g. What is the range of this function? Is the practical range different from the theoretical range?

The practical and theoretical ranges are the same because the function is periodic. $9.75 \le d(t) \le 14.47$

Where you should go for more review on this concept: See the Daylight Problem (3/11/13 post) and the Sine and Cosine Functions Quiz and Quiz Review (see 3/11/13 post for key to review).

9.



a. Fill in the following information about the function graphed above.

Amplitude: 1 Period: $\frac{\pi}{2}$

Vertical Shift: 0 Frequency: 4

Reflected? (Y/N) Sine or Cosine?

Equation: y= $-\cos(4x)$

b. Use your equation to find y when $x = 14.5\pi$.

$y = -\cos(4 \cdot 14.5\pi) = -1$

c. Based on the function you wrote in part a, change its amplitude to 3 and shift it vertically 5 units up. What is the new function?

$y = -3\cos(4x) + 5$

d. What is the domain of the function you wrote in part c? What is the range?

Domain: $\mathbb{R}$

Range: $2 \le y \le 8$

5-3 5+3

Where you should go for more review on this concept: 3/11/13 post with Quiz Review Key and key to the Sine and Cosine Functions Notes should be helpful.

2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.
  c. Develop and use the law of sines and the law of cosines.

When you take the AFM Common Exam, you will receive a formula sheet with the following information:

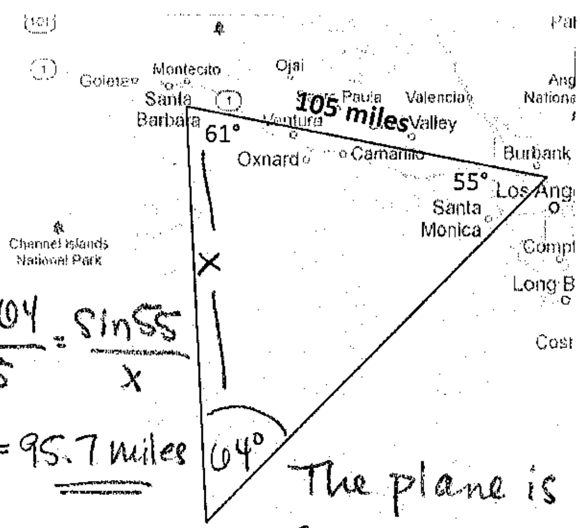
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

10. A commuter airplane, off course over the Pacific Ocean, reported experiencing mechanical problems in the middle of the night. The pilot sent two calls, one to the airport in Santa Barbara and one to the airport in Burbank, 105 miles away. Air traffic controllers at the two airports reported the angles shown in the diagram below. How far was the plane from the **closer** airport?

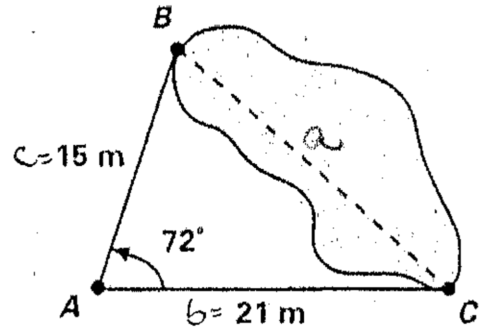


$$\frac{\sin 64}{105} = \frac{\sin 55}{x}$$

$x = 95.7$ miles

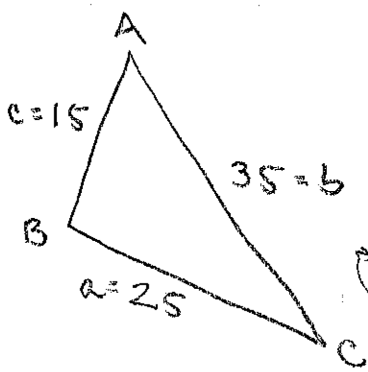
The plane is 95.7 miles from the Santa Barbara airport.

11. Based on the diagram below, calculate the length *BC* of the pond.



$$a^2 = 21^2 + 15^2 - 2 \cdot 21 \cdot 15 \cdot \cos 72°$$
$$a^2 = 471.3$$
$$a = 21.7 \text{ m}$$

12. A triangular region has sides 15 ft., 25 ft., and 35 ft. Find the measure of the region's largest angle.



$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

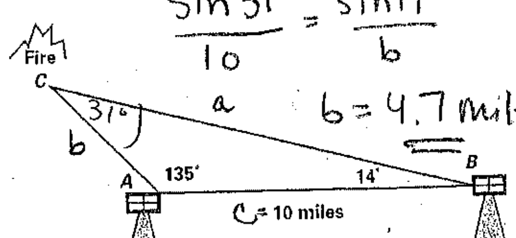
$$\cos B = \frac{35^2 - 25^2 - 15^2}{-2 \cdot 25 \cdot 15} = -.5$$

$$\cos^{-1}(-0.5) = 120°$$

13. How far away is the fire from observation tower A?

$$\frac{\sin 31°}{10} = \frac{\sin 14°}{b}$$

$b = 4.7$ miles



Where you should go for more review on this concept: See Law and Sines and Cosines Quiz and Quiz Review (see 2/13/13 post for key to review).

2.05 Use recursively-defined functions to model and solve problems.

a. Find the sum of a finite sequence.
b. Find the sum of an infinite sequence.
c. Determine whether a given series converges or diverges.
d. Translate between recursive and explicit representations.

When you take the AFM Common Exam, you will receive a formula sheet with the following information:

Arithmetic Sequence and Series

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Geometric Sequence and Series

$$a_n = a_1 \cdot r^{(n-1)}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}, \text{ where } r \neq 1$$

$$S = \frac{a_1}{1-r}, \text{ where } |r| < 1$$

14. Two prospective employers offer you a job. Calculate the total amount of money you would make in a month for each offer.

a. Company A offers a starting daily wage of 1 penny that doubles every day for 31 days.

(geometric)

$a_1 = 1$ $\quad r = 2 \quad n = 31$

$S_{31} = \frac{1(1-2^{31})}{1-2}$

$= 2147483647$ pennies

$= \$21,474,836.47$

b. Company B offers a starting daily wage of \$1000 that increases by \$10,000 every day for 31 days.

(arithmetic)

$a_1 = \$1000$

$d = \$10,000$

$n = 31$

$a_{31} = 1000 + 30 \cdot 10,000$

$= \$301,000$

$S_{31} = \frac{31}{2}(1000 + 301,000)$

$= \$4,681,000$

Which offer should you take? A!

15. A theater has 25 seats in the first row, 27 in the second row, 29 in the third row, and so on. (arithmetic)

a. How many seats are in the 50^{th} row?

$a_1 = 25$

$d = 2$

$n = 50$

$a_{50} = 25 + (50-1) \cdot 2$

$= 123$ seats

b. How many seats are in the entire theater?

$S_{50} = \frac{50}{2}(25 + 123)$

$= 3700$ seats

c. If the theater makes \$15 per seat, how much money should they expect to make at a sold-out show?

3700 seats $\times$ \$15 = \$55,500

16. Tell whether the following geometric series converge or diverge. Explain your reasoning. **If the series converges, find the sum.**

a. $$\sum_{n=1}^{\infty} \underset{a_1}{2}(\underset{r}{0.15})^{n-1}$$

converges ($|0.15| < 1$)

$S = \frac{2}{1-0.15} = 2.353$

b. $$\sum_{n=1}^{\infty} 0.6(3)^{n-1}$$

diverges ($|3| > 1$)

c. $$\sum_{n=1}^{\infty} 0.8^{n-1}$$

converges ($|0.8| < 1$)

$S = \frac{1}{1-0.8} = 5$

Where you should go for more review on this concept: See Sequences and Series Test and Test Review (see 1/17/13 post for key to review). Other posts from early January have more practice on sequences and series. Make sure you review going back and forth between the recursive and explicit versions of the formulas.